Econ 802

Final Exam

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Read each question carefully and try to use all of the information provided. All questions have equal weight. If something is unclear, please ask.

1. A firm has a strictly convex production possibilities set $Y \subset R^n$. For a production plan $y = (y_1 \,.\,.\, y_n) \in Y$, outputs are indicated by positive numbers and inputs are indicated by negative numbers.

(a) Prove that if the firm's profit maximization problem has a solution for the price vector $p > 0$, this solution is unique.

(b) Prove that if the firm's profit maximization problem has a unique solution $y(p)$ for each $p > 0$, the profit function $\pi(p) = py(p)$ is convex.

(c) In period $t = 1$, the price vector is $p^1 > 0$ and the production plan is y^1. In period $t = 2$, the price vector is $p^2 > 0$ and the production plan is y^2. Let $\Delta p = p^2 - p^1$ and $\Delta y = y^2 - y^1$. Prove that $\Delta p \Delta y \geq 0$ and interpret this result in words.

2. Consider the short run cost function $c(w, y) = w_f x_f + w_v x_v$ where $w = (w_1 \,.\,.\, w_n) = (w_f, w_v) > 0$. The subvector w_f gives the prices of the fixed inputs $x_f > 0$ and the subvector w_v gives the prices of the variable inputs $x_v \geq 0$ (the x_v inputs are chosen optimally). The output $y > 0$ is a scalar and the production function is $y = f(x_f, x_v)$.

(a) Use calculus to prove that when short run average cost is a <u>decreasing</u> function of output, marginal cost is <u>less</u> than average cost, and when short run average cost is an <u>increasing</u> function of output, marginal cost is <u>greater</u> than average cost.

(b) Suppose the production function is $y = \sum_{i = 1 \,.\,.\, n} a_i x_i$ where all $a_i > 0$. For a given $y > 0$, use Kuhn-Tucker multipliers to describe the set of variable inputs that could be used at a positive level, and the set of variable inputs that would definitely be equal to zero. <u>Note</u>: assume that the fixed inputs are not large enough to produce y, so variable inputs must be used.

(c) For the production function from (b), draw a graph with output on the horizontal axis and show the <u>short run</u> curves for average fixed cost, average variable cost, average total cost, and marginal cost. On a separate graph, indicate the shape of the <u>long run</u> average and marginal cost curves. Briefly justify your answers.

3. Consider the utility function $u = a \ln x_1 + b \ln x_2$ where $a > 0$ and $b > 0$.

(a) Compute the Marshallian demands $x(p, m)$ and the indirect utility function $v(p, m)$.

(b) Compute the Hicksian demands $h(p, u)$ and the expenditure function $e(p, u)$.

(c) Suppose a and b are both multiplied by the same constant $t > 0$. Does this affect the Marshallian demands? The Hicksian demands? Give an economic interpretation.

4. Each consumer $i = 1 \ldots n$ has a utility function $u_i = y_i + \ln x_i$ where y_i is leisure and x_i is widget consumption. Each consumer is endowed with T units of time but has a zero endowment of widgets. The budget constraint is $px_i = w(T - y_i)$ where p is the price of widgets and w is the wage rate.

(a) Find consumer i's demand for widgets $x_i(p, w)$ as a function of the prices. What is the market demand function for widgets $X(p, w)$?

(b) To produce z_j widgets in the long run, a firm j requires $c(z_j)$ units of labor, where $c(0) = 0$ and $c(z_j) = F + z_j^2$ for $z_j > 0$. $F > 0$ is the cost of setting up a firm. What is the supply function $z_j(p, w)$ for a typical firm j?

(c) Let labor be the numeraire so $w = 1$. Assume there is free entry and all firms are identical. Also assume that when the firms have zero profit, the number of firms is an integer. Using a graph, explain how you would derive the long run equilibrium price and quantity of widgets, as well as the equilibrium number of firms.

5. Asha and Ben have utility functions $u_A = \min \{x_{A1}, x_{A2}\}$ and $u_B = \min \{x_{B1}, x_{B2}\}$. The aggregate endowments are $w_1 = 3$ and $w_2 = 2$. There is no production.

(a) Draw an Edgeworth box and label the axes. Show a few indifference curves for each person and show the set of Pareto efficient points. Explain your reasoning.

(b) Suppose the individual endowments are $w_A = (1, 1)$ and $w_B = (2, 1)$. Derive the aggregate excess demand functions $z_1(p)$ and $z_2(p)$. If supply equals demand for good 1, what is true about p_2? If supply equals demand for good 2, what is true about p_1? Explain the reason for these unusual results.

(c) Assume the individual endowments are the same as in (b) and use an Edgeworth box to determine whether a Walrasian equilibrium (WE) exists. If there is a WE, describe an equilibrium price vector and allocation. If there is no WE, say why not. In either case, give a detailed justification for your answer.